



POSTAL BOOK PACKAGE 2027

ELECTRONICS ENGINEERING

CONVENTIONAL PRACTICE SETS **VOLUME - II**

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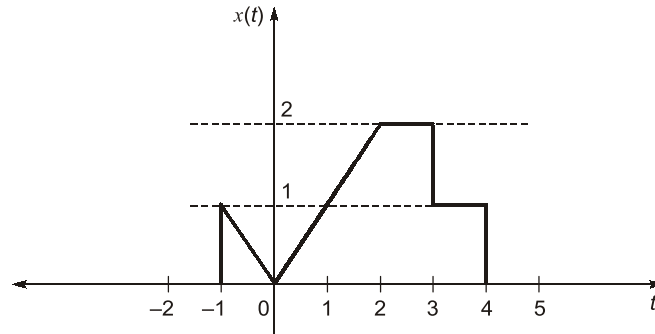
SIGNALS AND SYSTEMS

CONVENTIONAL PRACTICE SETS

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Continuous Time Signal & System

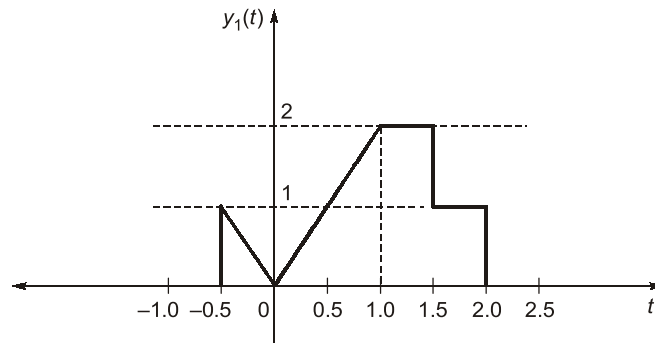
Q1 For the given signal $x(t)$ as shown below, sketch the following signals.



(a) $y_1(t) = x(2t)$ (b) $y_2(t) = x(2t + 4)$ (c) $y_3(t) = x\left(\frac{t}{2} + 2\right)$

Solution:

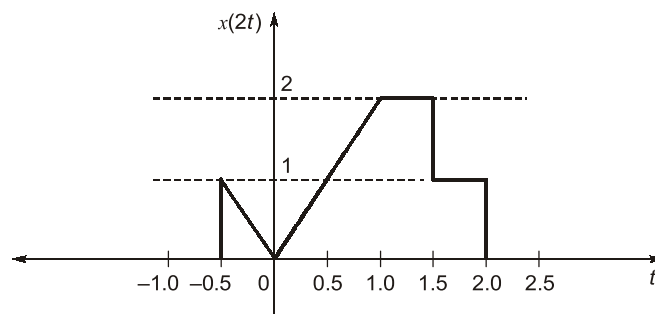
(a) We have to sketch, $y_1(t) = x(2t)$ since, $y_1(t)$ is a 2 times slowed or compressed version of $x(t)$ in time domain. So, the curve is;



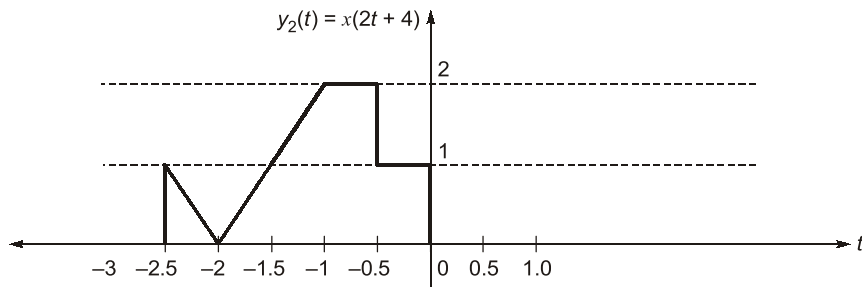
(b) We have to sketch, $y_2(t) = x(2t + 4)$ i.e. $y_2(t) = x[2(t + 2)]$.

So, we can say $y_2(t)$ is the 2 times compressed version in time of a signal which is an advance shift of 2 unit of $x(t)$.

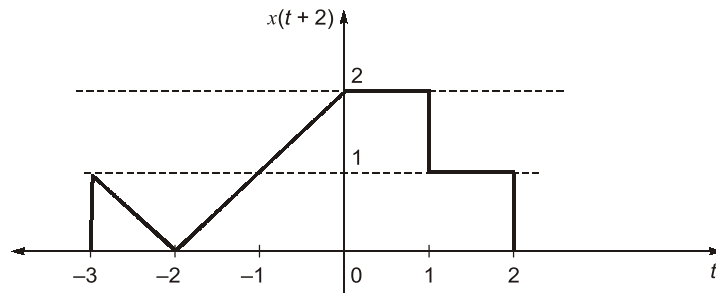
At first we sketch the following:



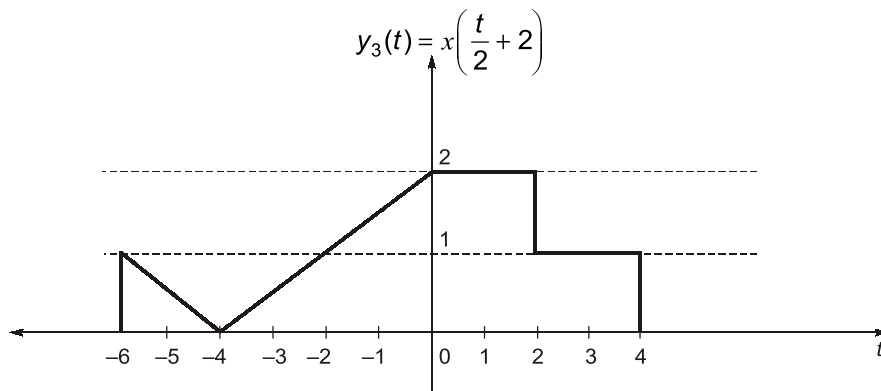
Again, we have to sketch this $x(2t)$ for an advance shift of 2 units means shift the above curve 2 unit in left side as below:



(c) We have to sketch, $y_3(t) = x(t/2 + 2)$. For this firstly, we shift 2 unit in advance shift of $x(t)$ and then expanded this signal $x(t + 2)$, by $1/(1/2) = 2$ units of the signal.



Now finally we have to sketch, $y_3(t) = x\left(\frac{t}{2} + 2\right)$ as below:



Q2 For the signals $y_1(t)$ and $y_2(t)$ shown below. Draw the differentiation of the signals and find the equations of differentiated signals.

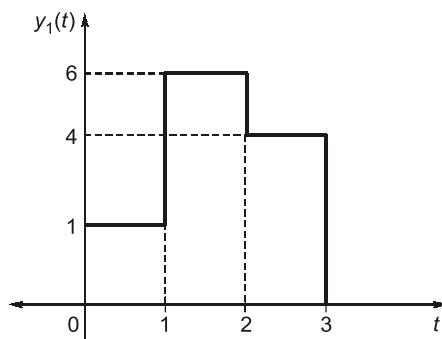


Fig. (a)

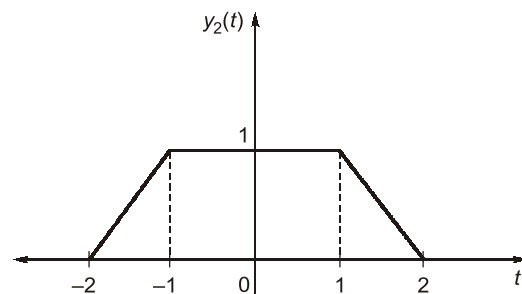


Fig. (b)

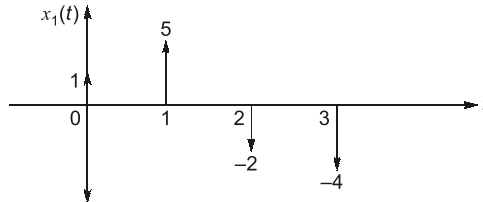
Solution:

For figure (a), we have to differentiate the signal $y_1(t)$.

Let,
$$\frac{dy_1(t)}{dt} = x_1(t) \quad \dots(i)$$

Given,
$$y_1(t) = u(t) + 5u(t-1) - 2u(t-2) - 4u(t-3)$$

So, from equation (i),
$$x_1(t) = \frac{dy_1(t)}{dt} = \delta(t) + 5\delta(t-1) - 2\delta(t-2) - 4\delta(t-3)$$



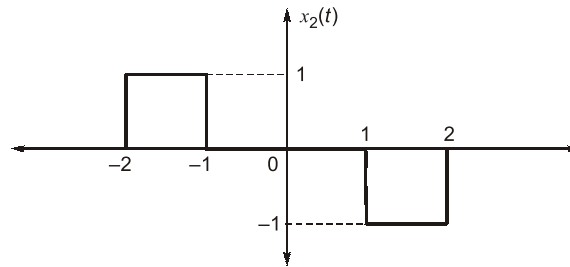
For figure (b), we have to differentiate the signal $y_2(t)$.

Let,
$$\frac{dy_2(t)}{dt} = x_2(t) \quad \dots(ii)$$

Given,
$$y_2(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$$

(where $r(t)$ represents the ramp function)

So, from equation (ii),
$$x_2(t) = \frac{dy_2(t)}{dt} = u(t+2) - u(t+1) - u(t-1) + u(t-2)$$



Q3 Show that the signal, $S(t) = t^{-1/4} u(t-1)$ is neither an energy nor a power signal.

Solution:

For an arbitrary continuous-time signal $s(t)$, the normalized energy content ' E ' of $S(t)$ is defined as,

$$E = \int_{-\infty}^{\infty} |S(t)|^2 dt \quad \dots(i)$$

or
$$E = \int_{-\infty}^{\infty} |t^{-1/4} u(t-1)|^2 dt$$

Since,
$$u(t-1) = \begin{cases} 1, & t > 1 \\ 0, & t < 1 \end{cases}$$

$\therefore E = \int_1^{\infty} |t^{-1/4}|^2 dt = \int_1^{\infty} [t]^{-1/2} dt = [2[t]^{1/2}]_1^{\infty}$

$\therefore E = \infty$

Now, the normalized average power ' P ' of $S(t)$ is defined as,

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |S(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |t^{-1/4} u(t-1)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_1^T (t^{-1/4})^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_1^T t^{-1/2} dt$$

$$\therefore P = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[2t^{1/2} \right]_1^T = \lim_{T \rightarrow \infty} \left[\frac{T^{1/2} - 1}{T} \right] = \lim_{T \rightarrow \infty} \left[\frac{1 - 1/\sqrt{T}}{\sqrt{T}} \right] = 0$$

$$\therefore P = 0$$

Here average power $P \rightarrow 0$, when total energy $E \rightarrow \infty$, which means that the condition $0 < E < \infty$ is not satisfied. Hence signal, $S(t) = t^{-1/4} u(t-1)$ is not an energy signal. Also, when $E \rightarrow \infty$, the value of $P \rightarrow 0$ which means that the condition $0 < P < \infty$ is not satisfied. Therefore, signal $S(t) = t^{-1/4} u(t-1)$ is not a power signal.

Thus, we can say that the given signal, $S(t) = t^{-1/4} u(t-1)$ is neither an energy signal nor a power signal.

Proved.

Q4 Consider a continuous-time system with input $x(t)$ and output $y(t)$ given by

$$y(t) = x(t) \cos(t)$$

Check whether the is

(a) linear (b) time-invariant

Solution:

$$y(t) = x(t) \cos(t)$$

(a) To check linearity,

$$y_1(t) = x_1(t) \cos(t)$$

[$y_1(t)$ is output for $x_1(t)$]

$$y_2(t) = x_2(t) \cos(t)$$

[$y_2(t)$ is output for $x_2(t)$]

So the output for $(x_1(t) + x_2(t))$ will be

$$\begin{aligned} y(t) &= [x_1(t) + x_2(t)] \cos(t) \\ &= y_1(t) + y_2(t) \end{aligned}$$

So the system is linear to check time invariance.

(b) The delayed output, $y(t - t_0) = x(t - t_0) \cos(t - t_0)$

The output for delayed input,

$$y(t, t_0) = x(t - t_0) \cos(t)$$

Since,

$$y(t - t_0) \neq y(t, t_0)$$

System is time varying.

Q5 Find out whether the system is stable/causal. If the impulse response is given by, $h(t) = e^{-6|t|}$.

Solution:

As given that,

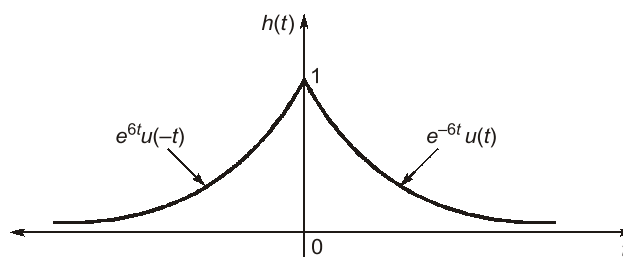
$$h(t) = e^{-6|t|}$$

\therefore

$$h(t) = e^{-6t} \cdot u(t) + e^{6t} \cdot u(-t)$$

...(i)

Here we see that, $h(t) \neq 0$ for $t < 0$ so, the given system is not **causal**.



For checking the system to be “**BIBO stable**”, we know that,

$$\sum_{t=-\infty}^{\infty} h(\tau) d\tau < \infty$$

...(ii)

$$\begin{aligned} \text{L.H.S of equation (ii)} &= \sum_{t=-\infty}^{\infty} h(\tau) d\tau \\ &= \sum_{t=-\infty}^{\infty} [e^{-6\tau} u(t) d\tau + e^{6\tau} u(-\tau) d\tau] = \int_{t=0}^{\infty} e^{-6\tau} d\tau + \int_{-\infty}^0 e^{6\tau} d\tau \\ &= -\frac{1}{6} [e^{-6\tau}]_0^{\infty} + \frac{1}{6} [e^{6\tau}]_{-\infty}^0 = -\frac{1}{6}(0-1) + \frac{1}{6}(1-0) \end{aligned}$$

L.H.S of equation (ii) = $\frac{1}{3} < \infty$

So, the system is "BIBO Stable".

Q6 A continuous time LTI system is described by:

$$y(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau) d\tau$$

Find the impulse response of the system. Is the system casual?

Solution:

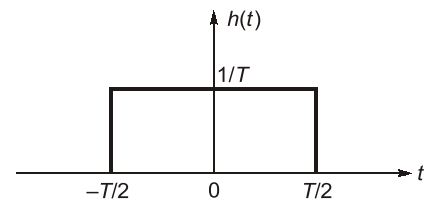
1st Method:
$$y(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} x(\tau) d\tau \quad \dots(i)$$

Let the impulse response be $h(t)$,

$$\therefore y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau) d\tau \quad (ii)$$

Comparing equation (i) and (ii) we get,

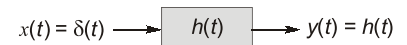
$$h(t) = \begin{cases} \frac{1}{T} & -\frac{T}{2} < t < \frac{T}{2} \\ 0 & \text{Otherwise} \end{cases}$$



2nd method:

$$\therefore h(t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} \delta(t) dt$$

$$\Rightarrow h(t) = u\left(t + \frac{T}{2}\right) - u\left(t - \frac{T}{2}\right)$$



The system is not casual as we can clearly see from equation (i) that for calculation of $y(t)$ at any time t , we require future values of input $x(t)$.

Another way to see this is that $h(t)$ is not zero for $t < 0$, which is the basic requirement for any casual system.

Q7 Show that the following properties holds good for the derivative of $\delta(t)$.

(a)
$$\int_{-\infty}^{\infty} \phi(t) \delta'(t) dt = -\phi'(0)$$

where, $\phi'(0) = \left. \frac{d\phi(t)}{dt} \right|_{t=0}$

(b) $t\delta'(t) = -\delta(t)$

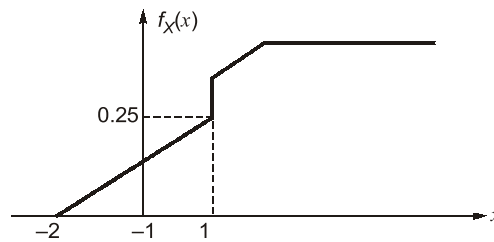
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Theory of Random Variable and Noise

Q1 Define PDF and summarise its important properties. Also calculate the probability of outcome of a Random Variable (RV) X having $X \leq 1$ for the following PDF curve of RV as shown.



Solution:

Probability density function specifies the probability of a random variable taking a particular value.

The Probability Density Function (PDF) which is generally denoted by $f_X(x)$ or $P_X(x)$ or $p_X(x)$ is defined in terms of the Cumulative Distribution Function (CDF) $F_X(x)$ as,

$$\text{PDF} = f_X(x) = \frac{d}{dx} F_X(x) \quad \dots(i)$$

The PDF has the following properties:

(i) $f_X(x) \geq 0$ for all x

This results from the fact that probability cannot be negative. Also, $F_X(x)$ increases monotonically, as x increases, more outcomes are included in the prob. of occurrence represented by $F_X(x)$.

(ii) Area under the PDF curve is always equal to unity.

i.e.
$$\int_{-\infty}^{\infty} f_X(x) dx = 1$$

(iii) The CDF is obtained by the result

$$\text{CDF} = \int_{-\infty}^x f_X(x) dx$$

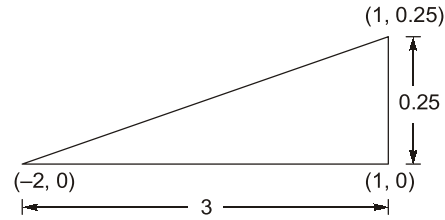
(iv) Probability of occurrence of the value of random variable between the limits of x_1 and x_2 is given by,

$$P(x_1 < X < x_2) = \int_{x_1}^{x_2} f_X(x) dx$$

Now consider the given PDF curve, since we have to find $P(x \leq 1)$ so,

Equation for the PDF curve for $x \leq 1$ is,

$$f_X(x) = \left(\frac{1}{12}x + \frac{1}{6} \right)$$



Now, $P(x \leq 1)$

$$= P(-2 < x < 1) = \int_{-2}^1 \left(\frac{1}{12}x + \frac{1}{6} \right) dx = \left[\frac{1}{12} \cdot \frac{x^2}{2} + \frac{1}{6}x \right]_{-2}^1 = \frac{3}{8}$$

$$\therefore P(x \leq 1) = \frac{3}{8}$$

Q2 Find the cumulative distribution function $F(x)$ corresponding to the PDF $f(x) = \frac{1}{\pi(1+x^2)}$, $-\infty < x < \infty$.

Solution:

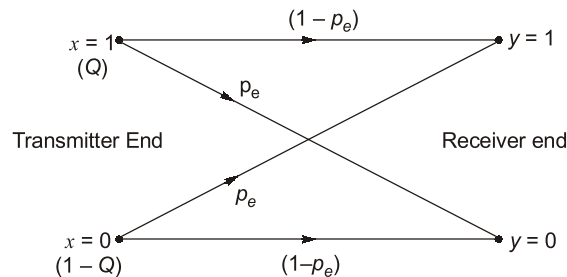
Given

$$f(x) = \frac{1}{\pi(1+x^2)}, \quad -\infty < x < \infty$$

$$F(x) = P(X \leq x)$$

$$= \int_{-\infty}^x f(x) dx = \frac{1}{\pi} \int_{-\infty}^x \frac{dx}{1+x^2} = \frac{1}{\pi} [\tan^{-1} x]_{-\infty}^x = \frac{1}{\pi} \left(\frac{\pi}{2} + \tan^{-1} x \right)$$

Q3 A BSC (Binary Symmetric Channel) error probability is P_e . The probability of transmitting '1' is Q , and that of transmitting '0' is $(1-Q)$ as in figure below. Calculate the probabilities of receiving 1 and 0 at the receiver?



Solution:

If x and y are the transmitted digit and the received digit respectively, then for a BSC,

$$P_{y|x}(0|1) = P_{y|x}(1|0) = P_e$$

$$P_{y|x}(0|0) = P_{y|x}(1|1) = 1 - P_e$$

Also,

$$P_x(1) = Q \text{ and } P_x(0) = 1 - Q$$

We have to find, $P_y(1)$ and $P_y(0) = ?$

$$\therefore P_y(1) = P_x(0) P_{y|x}(1|0) + P_x(1) P_{y|x}(1|1) = (1-Q)P_e + Q(1-P_e)$$

also,

$$P_y(0) = P_x(0) P_{y|x}(0|0) + P_x(1) P_{y|x}(0|1) = (1-Q)(1-P_e) + QP_e$$

Q4 For the triangular distribution

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2-x, & 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and variance.

Solution:

$$\begin{aligned} \text{Mean} = E(X) &= \int_{-\infty}^{\infty} xf(x)dx = \int_0^1 x \cdot x dx + \int_1^2 x(2-x)dx = \int_0^1 x^2 dx + \int_1^2 (2x - x^2)dx \\ &= \left[\frac{x^3}{3} \right]_0^1 + \left[2\left(\frac{x^2}{2}\right) - \frac{x^3}{3} \right]_1^2 \\ &= \frac{1}{3} + \left[\left(4 - \frac{8}{3}\right) - \left(1 - \frac{1}{3}\right) \right] = \frac{1}{3} + \frac{4}{3} - \frac{2}{3} = 1 \\ E(X^2) &= \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^1 x^2 x dx + \int_1^2 x^2(2-x)dx \\ &= \int_0^1 x^3 dx + \int_1^2 (2x^2 - x^3)dx = \left[\frac{x^4}{4} \right]_0^1 + \left[2\left(\frac{x^3}{3}\right) - \frac{x^4}{4} \right]_1^2 \\ &= \frac{1}{4} + \left[\left(\frac{16}{3} - \frac{16}{4}\right) - \left(\frac{2}{3} - \frac{1}{4}\right) \right] = \frac{1}{4} + \frac{16}{3} - 4 - \frac{2}{3} + \frac{1}{4} = \frac{7}{6} \\ \text{Var}(X) &= E(X^2) - E(X)^2 = \frac{7}{6} - (1)^2 = \frac{1}{6} \end{aligned}$$

Q5 The joint density function of two continuous random variables is given by

$$f(x, y) = \begin{cases} xy/8, & 0 < x < 2, 1 < y < 3 \\ 0, & \text{otherwise} \end{cases}$$

Find (a) $E(X)$, (b) $E(Y)$ and (c) $E(2X + 2Y)$.

Solution:

$$\begin{aligned} \text{(a)} \quad E(X) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xf(x, y) dx dy = \int_{x=0}^2 \int_{y=1}^3 x(xy/8) dx dy = \frac{4}{3} \\ \text{(b)} \quad E(Y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} yf(x, y) dx dy = \int_{x=0}^2 \int_{y=1}^3 y(xy/8) dx dy = \frac{13}{6} \\ \text{(c)} \quad E(2X + 3Y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (2x + 3y) dx dy = \int_{x=0}^2 \int_{y=1}^3 (2x + 3y)(xy/8) dx dy = \frac{55}{6} \end{aligned}$$

Q6 A WSS random process $x(t)$ is applied to the input of an LTI system with impulse response

$$h(t) = 3e^{-2t} u(t)$$

Find the mean value of the output $y(t)$ of the system, if $E[x(t)] = 2$. Here $E[\cdot]$ denotes the expectation operator.

Solution:

The output $y(t)$ is the convolution of the input $x(t)$ and the impulse response $h(t)$.

$$y(t) = \int_{-\infty}^{\infty} h(\tau) \cdot x(t - \tau) \cdot d\tau$$

\therefore

$$E[y(t)] = \int_{-\infty}^{\infty} h(\tau) \cdot E[x(t - \tau)] \cdot d\tau$$

$$E[y(t)] = H(0) \times E[x(t)]$$

$$E[y(t)] = E[x(t)] \cdot H(0)$$

where, $H(0) = H(\omega)|_{\omega=0}$ and $H(\omega) =$ Fourier transform of $h(t)$

Given $E[x(t)] = 2$,

$$h(t) = 3e^{-2t} u(t)$$

Taking Fourier transform,

$$H(\omega) = \frac{3}{2 + j\omega} \Rightarrow H(0) = \frac{3}{2}$$

$$E[y(t)] = 2 \times \frac{3}{2} = 3$$

Q7 Suppose that two signals $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(0, T)$. A sample function $n(t)$ of a zero-mean white noise process is correlated with $s_1(t)$ and $s_2(t)$ separately, to yield the following variables:

$$n_1 = \int_0^T s_1(t) n(t) dt \quad \text{and} \quad n_2 = \int_0^T s_2(t) n(t) dt$$

Prove that n_1 and n_2 are orthogonal.

Solution:

$$\begin{aligned} E[n_1 n_2] &= E \left[\int_0^T s_1(u) n(u) du \int_0^T s_2(v) n(v) dv \right] \\ &= \int_0^T \int_0^T s_1(u) s_2(v) E[n(u) n(v)] du dv \end{aligned}$$

$n(t)$ is a white noise process.

$$\text{So,} \quad R_n(\tau) = \frac{N_0}{2} \delta(\tau)$$

$$E[n(u) n(v)] = \frac{N_0}{2} \delta(u - v)$$

$$\text{Hence,} \quad E[n_1 n_2] = \frac{N_0}{2} \int_0^T \int_0^T s_1(u) s_2(v) \delta(u - v) du dv$$

$$= \frac{N_0}{2} \int_0^T s_1(u) s_2(u) du$$

$$= 0 \quad \because s_1(t) \text{ and } s_2(t) \text{ are orthogonal over } (0, T)$$

$E[n_1 n_2] = 0$. So, n_1 and n_2 are also orthogonal.

Q8 Find the time autocorrelation function of the signal $g(t) = e^{-at} u(t)$ and from this obtain the energy spectral density (ESD) of $g(t)$.

Solution:

Auto correlation function,

$$\begin{aligned} R_x(\tau) &= \int_{-\infty}^{\infty} g(t) g(t - \tau) dt = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-a(t-\tau)} u(t - \tau) dt \\ &= \int_{\tau}^{\infty} e^{-2at} e^{a\tau} dt = e^{a\tau} \int_{\tau}^{\infty} e^{-2at} dt = \frac{e^{a\tau}}{-2a} \left[e^{-2at} \right]_{\tau}^{\infty} = \frac{e^{a\tau}}{2a} e^{-2a\tau} = \frac{e^{-a\tau}}{2a} \end{aligned}$$

Similar process is valid for negative side because for real $g(t)$, $R_x(\tau)$ is even function of time

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Vector Analysis

Q1 For a vector field \vec{A} , show explicitly that $\nabla \cdot \nabla \times \vec{A} = 0$; that is, the divergence of the curl of any vector field is zero.

Solution:

$$\begin{aligned}\nabla \cdot \nabla \times \vec{A} &= \frac{\partial}{\partial x} \cdot \hat{a}_x + \frac{\partial}{\partial y} \cdot \hat{a}_y + \frac{\partial}{\partial z} \cdot \hat{a}_z \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \\ &= \frac{\partial}{\partial x} \cdot \hat{a}_x + \frac{\partial}{\partial y} \cdot \hat{a}_y + \frac{\partial}{\partial z} \cdot \hat{a}_z \left[\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{a}_x - \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) \hat{a}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{a}_z \right] \\ &= \frac{\partial}{\partial x} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) - \frac{\partial}{\partial y} \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) + \frac{\partial}{\partial z} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\ &= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} - \frac{\partial^2 A_z}{\partial y \partial x} + \frac{\partial^2 A_x}{\partial y \partial z} + \frac{\partial^2 A_y}{\partial z \partial x} - \frac{\partial^2 A_x}{\partial z \partial y} = 0\end{aligned}$$

Because $\frac{\partial^2 A_z}{\partial x \partial y} = \frac{\partial^2 A_z}{\partial y \partial x}$ and so on.

Q2 If $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$, then show that the vector $\vec{A} = xz \hat{a}_x + z^2 \hat{a}_y + yz \hat{a}_z$ satisfy this vector identity.

Solution:

We have to verify, $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$... (i)

Taking L.H.S.

$$\begin{aligned}\nabla \times \vec{A} &= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & z^2 & yz \end{vmatrix} \\ &= \left(\frac{\partial}{\partial x} z^2 - \frac{\partial}{\partial y} xz \right) \hat{a}_z + \left(\frac{\partial}{\partial y} (yz) - \frac{\partial}{\partial z} (z^2) \right) \hat{a}_x - \left(\frac{\partial}{\partial x} yz - \frac{\partial}{\partial z} xz \right) \hat{a}_y \\ &= (z - 2z) \hat{a}_x + x \hat{a}_y = -z \hat{a}_x + x \hat{a}_y \quad \dots \text{(ii)}\end{aligned}$$

\therefore

$$\begin{aligned}\nabla \times (\nabla \times \vec{A}) &= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -z & x & 0 \end{vmatrix} \\ &= 0 \cdot \hat{a}_x - (+1) \hat{a}_y + \hat{a}_z = -\hat{a}_y + \hat{a}_z \quad \dots \text{(iii)}\end{aligned}$$

now considering R.H.S

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z$$

$$\nabla \cdot \vec{A} = \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial y}(z^2) + \frac{\partial}{\partial z} yz = z + y$$

$$\begin{aligned} \nabla(\nabla \cdot \vec{A}) &= \frac{\partial}{\partial x}(\nabla \cdot \vec{A})\hat{a}_x + \frac{\partial}{\partial y}(\nabla \cdot \vec{A})\hat{a}_y + \frac{\partial}{\partial z}(\nabla \cdot \vec{A})\hat{a}_z = \frac{\partial}{\partial x}(z+y)\hat{a}_x + \frac{\partial}{\partial y}(z+y)\hat{a}_y + \frac{\partial}{\partial z}(z+y)\hat{a}_z \\ &= \hat{a}_y + \hat{a}_z \end{aligned} \quad \dots(\text{iv})$$

$$\nabla^2 \vec{A} = \nabla^2 A_x \hat{a}_x + \nabla^2 A_y \hat{a}_y + \nabla^2 A_z \hat{a}_z$$

$$\begin{aligned} \text{and } \nabla^2 \vec{A} &= \left[\frac{\partial^2}{\partial x^2}(xz) + \frac{\partial^2}{\partial y^2}(xz) + \frac{\partial^2}{\partial z^2}(xz) \right] \hat{a}_x + \left[\frac{\partial^2}{\partial x^2}(z^2) + \frac{\partial^2}{\partial y^2}(z^2) + \frac{\partial^2}{\partial z^2}(z^2) \right] \hat{a}_y + \left[\frac{\partial^2}{\partial x^2}(yz) + \frac{\partial^2}{\partial y^2}(yz) + \frac{\partial^2}{\partial z^2}(yz) \right] \hat{a}_z \\ \nabla^2 \vec{A} &= 2\hat{a}_y \end{aligned} \quad \dots(\text{v})$$

From equation (iv) and (v), we get,

$$\nabla \cdot (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \hat{a}_y + \hat{a}_z - 2\hat{a}_y = \hat{a}_z - \hat{a}_y \quad \dots(\text{vi})$$

∴ From equation (iii) and (vi),

$$\text{RHS} = \text{LHS} = \hat{a}_z - \hat{a}_y$$

Hence Proved.

Q3 If $\vec{F} = x^2 y \hat{a}_x - 2z \hat{a}_y + (3z^2 + xy) \hat{a}_z$, find $\nabla \times [\nabla \cdot (\nabla \cdot \vec{F})]$.

Solution:

$$\text{Given} \quad \vec{F} = x^2 y \hat{a}_x - 2z \hat{a}_y + (3z^2 + xy) \hat{a}_z$$

Let us calculate $\nabla \times [\nabla \cdot (\nabla \cdot \vec{F})]$ step by step.

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(x^2 y) - \frac{\partial}{\partial y}(2z) + \frac{\partial}{\partial z}(3z^2 + xy) = 2xy + 6z$$

$$\nabla \cdot (\nabla \cdot \vec{F}) = \nabla \cdot (2xy + 6z)$$

$$= \frac{\partial}{\partial x}(2xy + 6z)\hat{a}_x + \frac{\partial}{\partial y}(2xy + 6z)\hat{a}_y + \frac{\partial}{\partial z}(2xy + 6z)\hat{a}_z = 2y\hat{a}_x + 2x\hat{a}_y + 6\hat{a}_z$$

$$\nabla \times [\nabla \cdot (\nabla \cdot \vec{F})] = \nabla \times (2y\hat{a}_x + 2x\hat{a}_y + 6\hat{a}_z)$$

$$= \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2y & 2x & 6 \end{bmatrix} = \left(\frac{\partial(6)}{\partial y} - \frac{\partial(2x)}{\partial z} \right) \hat{a}_x - \left(\frac{\partial(6)}{\partial x} - \frac{\partial(2y)}{\partial z} \right) \hat{a}_y + \left(\frac{\partial(2x)}{\partial x} - \frac{\partial(2y)}{\partial y} \right) \hat{a}_z$$

$$= 0\hat{a}_x - 0\hat{a}_y + (2 - 2)\hat{a}_z = 0$$

Note that $\nabla \times \nabla \cdot \vec{F} = 0$

Q4 E and F are vector fields given by $E = 2x \hat{a}_x + \hat{a}_y + yz \hat{a}_z$ and $F = xy \hat{a}_x - y^2 \hat{a}_y + xy z \hat{a}_z$. Determine

(a) $|E|$ at $(1, 2, 3)$

(b) The component of E along F at $(1, 2, 3)$

(c) A vector perpendicular to both E and F at $(0, 1, -3)$ whose magnitude is unity.

Solution:

(a) $\vec{E} = 2x \hat{a}_x + \hat{a}_y + yz \hat{a}_z$; At point (1, 2, 3) $\Rightarrow \vec{E} = 2\hat{a}_x + \hat{a}_y + 6\hat{a}_z$

$$|\vec{E}| = \sqrt{2^2 + 1^2 + 6^2} = \sqrt{41} = 6.403$$

(b) $\vec{F} = xy \hat{a}_x - y^2 \hat{a}_y + xyz \hat{a}_z$; At (1, 2, 3), $\vec{F} = 2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z$

\therefore The component of \vec{E} along \vec{F}

$$E_F = (\vec{E} \cdot \hat{a}_F) \hat{a}_F \quad \left[\hat{a}_F = \frac{\vec{F}}{|\vec{F}|} \right]$$

$$= \frac{(\vec{E} \cdot \vec{F})}{|\vec{F}|^2} \vec{F} = \frac{36}{56} (2\hat{a}_x - 4\hat{a}_y + 6\hat{a}_z) = 1.286\hat{a}_x - 2.571\hat{a}_y + 3.857\hat{a}_z$$

(c) At (0, 1, -3) $\vec{E} = 0\hat{a}_x + \hat{a}_y - 3\hat{a}_z$

$$\vec{F} = 0\hat{a}_x - \hat{a}_y + 0\hat{a}_z$$

$$E \times \vec{F} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ 0 & 1 & -3 \\ 0 & -1 & 0 \end{vmatrix} = -3\hat{a}_x + 0\hat{a}_y + 0\hat{a}_z$$

$$a_{E \times F} = \pm \frac{\vec{E} \times \vec{F}}{|\vec{E} \times \vec{F}|} = \pm \hat{a}_x$$

Q5 Let $\vec{H} = 5\rho \sin\phi \hat{a}_\rho - \rho z \cos\phi \hat{a}_\phi + 2\rho \hat{a}_z$ A/m. At point $P(2, 30^\circ, -1)$ find:

(a) a unit vector along \vec{H} .

(b) the component of \vec{H} parallel to \hat{a}_x .

(c) the component of \vec{H} normal to $\rho = 2$.

(d) the component of \vec{H} tangential to $\phi = 30^\circ$.

Solution:

At P , $\rho = 2, \phi = 30^\circ, z = -1$

$$\vec{H} = 10 \sin 30^\circ \hat{a}_\rho + 2 \cos 30^\circ \hat{a}_\phi + 4 \hat{a}_z = 5\hat{a}_\rho + 1.732 \hat{a}_\phi + 4\hat{a}_z \text{ A/m}$$

(a) Unit vector along \vec{H} ,

$$\hat{a}_H = \frac{5\hat{a}_\rho + 1.732 \hat{a}_\phi + 4\hat{a}_z}{\sqrt{5^2 + 1.732^2 + 4^2}} = 0.7538 \hat{a}_\rho + 0.2611 \hat{a}_\phi + 0.603 \hat{a}_z$$

(b) The relation between Cartesian coordinates (A_x, A_y, A_z) and cylindrical coordinates (A_ρ, A_ϕ, A_z) is given as:

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$H_x = H_\rho \cos\phi - H_\phi \sin\phi = 5\rho \sin\phi \cos\phi - \rho z \cos\phi \sin\phi$$

At P , $\rho = 5, \phi = 30^\circ, z = -1$

$$H_x = H_x \hat{a}_x = (25 \sin 30^\circ \cos 30^\circ + 5 \sin 30^\circ \cos 30^\circ) \hat{a}_x = 13 \hat{a}_x \text{ A/m}$$

(c) Normal to $\rho = 2$ is $\vec{H}_n = \vec{H}_\rho \hat{a}_\rho$

i.e. $\vec{H}_n = 0.7538 \hat{a}_\rho \text{ A/m}$

(d) Tangential to $\phi = 30^\circ$

$$H_t = H_\rho \cdot \hat{a}_\rho + H_z \hat{a}_z = 0.7538 \hat{a}_\rho + 0.603 \hat{a}_z \text{ A/m}$$

Q6 Find the rate at which the scalar function, $V = r^2 \sin 2\phi$, in cylindrical co-ordinates, increases in the direction of the vector $A = \hat{a}_r + \hat{a}_\phi$ at the point having co-ordinates $(2, \pi/4, 0)$.

Solution:

As we know that, Gradient is a vector that represents both the magnitude and the direction of maximum space rate of the increase of the scalar function i.e.

$$\text{grad } V = \nabla V = \frac{dV}{dn} \hat{a}_n \quad \dots(i)$$

But in cylindrical coordinate system, the grad V can be defined as,

$$\nabla V = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \phi} \hat{a}_\phi + \frac{\partial V}{\partial z} \hat{a}_z \quad \dots(ii)$$

For the case under consideration, the quantity required is,

$$\nabla V \cdot \hat{a}_A = \nabla V \cdot \frac{\vec{A}}{|\vec{A}|} \quad \text{at } (2, \pi/4, 0)$$

From equation (ii) we have,

$$\nabla V = \frac{\partial}{\partial r}(r^2 \sin 2\phi) \hat{a}_r + \frac{1}{r} \frac{\partial}{\partial \phi}(r^2 \sin 2\phi) \hat{a}_\phi + \frac{\partial}{\partial z}(r^2 \sin 2\phi) \hat{a}_z.$$

or,

$$\nabla V = 2r \sin 2\phi \hat{a}_r + 2r \cos 2\phi \hat{a}_\phi$$

Now,

$$\nabla V \cdot \vec{A} = (2r \sin 2\phi \hat{a}_r + 2r \cos 2\phi \hat{a}_\phi) \cdot (\hat{a}_r + \hat{a}_\phi)$$

or,

$$\nabla V \cdot \vec{A} = 2r \sin 2\phi + 2r \cos 2\phi \quad \dots(iii)$$

Also,

$$|\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

\therefore

$$\begin{aligned} \nabla V \cdot \hat{a}_A &= \frac{\nabla V \cdot \vec{A}}{|\vec{A}|} = \frac{1}{\sqrt{2}} [2r \sin 2\phi + 2r \cos 2\phi] \\ &= (\sqrt{2} r \sin 2\phi + \sqrt{2} r \cos 2\phi) \end{aligned}$$

$$\text{Now, } (\nabla V \cdot \hat{a}_A)_{\text{at}(2, \pi/4, 0)} = \sqrt{2} \times 2 \times \sin\left(2 \times \frac{\pi}{4}\right) + \sqrt{2} \times 2 \times \cos\left(2 \times \frac{\pi}{4}\right) = 2\sqrt{2}$$

Q7 Write down the Divergence theorem. An electric field at point P , expressed in cylindrical co-ordinate system is given by,

$$\vec{E} = 3r^2 \sin \phi \hat{a}_r + 2r^2 \cos \phi \hat{a}_\phi$$

Find the value of divergence of the field if the location of point ' P ' is given by $(9, 9, 9)$ in Cartesian co-ordinate system.

Solution:

Divergence Theorem: The divergence theorem states that, the volume integral of the divergence of a vector field ' A ', taken over any volume V is equal to the surface integral of A , taken over the closed surface that is

$$\int_V (\nabla \cdot \vec{A}) dV = \oint_S \vec{A} \cdot d\vec{s}$$

Given that,

$$\vec{E} = 3r^2 \sin \phi \hat{a}_r + 2r^2 \cos \phi \hat{a}_\phi$$

In cylindrical coordinate system,

$$\nabla \cdot \vec{E} = \frac{1}{r} \frac{\partial}{\partial r}(rE_r) + \frac{1}{r} \frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r}(3r^3 \sin \phi) + \frac{1}{r} \frac{\partial}{\partial \phi}(2r^2 \cos \phi)$$

$$= \frac{1}{r}(9r^2 \sin\phi) + \frac{1}{r}(2r^2(-\sin\phi)) = 9r\sin\phi - 2r\sin\phi$$

$$\nabla \cdot \vec{E} = 7r\sin\phi$$

Converting into Cartesian co-ordinates

$$r = \sqrt{x^2 + y^2}$$

and

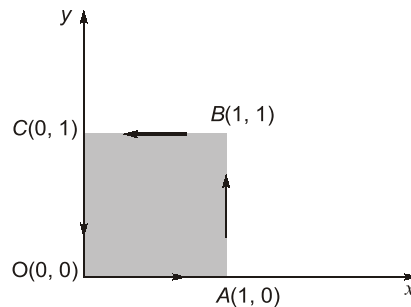
$$\sin\phi = \frac{y}{\sqrt{x^2 + y^2}}$$

\therefore

$$\nabla \cdot \vec{E} = 7\sqrt{x^2 + y^2} \left(\frac{y}{\sqrt{x^2 + y^2}} \right) = 7\sqrt{9^2 + 9^2} \cdot \frac{9}{\sqrt{9^2 + 9^2}} = 7 \times 9 = 63$$

$$\nabla \cdot \vec{E} = 63$$

- Q8** (a) Given vector $\vec{F} = x^2y\hat{a}_x + 2xy^2\hat{a}_y$, find circulation of \vec{F} along a closed path $OABC$ as shown in figure below:



- (b) Check the above result using Stoke's theorem. According to Stoke's theorem

$$\oint \vec{F} \cdot d\vec{l} = \int (\nabla \times \vec{F}) \cdot d\vec{s}$$

Solution:

- (a) In a closed path the circulation of vector \vec{F} is given as

$$\oint \vec{F} \cdot d\vec{l} = \int_{OA} + \int_{AB} + \int_{BC} + \int_{CO} \vec{F} \cdot d\vec{l}$$

Given $\vec{F} = x^2y\hat{a}_x + 2xy^2\hat{a}_y$; $d\vec{l} = dx\hat{a}_x + dy\hat{a}_y$; and $\vec{F} \cdot d\vec{l} = F_x dx + F_y dy = x^2y dx + 2xy^2 dy$

For path OA , $dy = 0$, $y = 0$, $F = 0$

and,

$$\int_{OA} \vec{F} \cdot d\vec{l} = \int_{OA} x^2y dx = 0$$

For path AB , $dx = 0$, $x = 1$, $F = y\hat{a}_x + 2y^2\hat{a}_y$

and

$$\int_{AB} \vec{F} \cdot d\vec{l} = \int_0^1 2y^2 dy = \frac{2y^3}{3} = \frac{2}{3}(1-0) = \frac{2}{3}$$

For path BC ,

$$dy = 0, \quad y = 1, \quad \vec{F} = x^2\hat{a}_x + 2x\hat{a}_y$$

and,

$$\int_{BC} \vec{F} \cdot d\vec{l} = \int_1^0 x^2 dx = \frac{x^3}{3} = \frac{-1}{3}$$

DIGITAL CIRCUITS

CONVENTIONAL PRACTICE SETS

Page No. 316 - 385

Number Systems and Codes

- Q1** (i) Convert octal 756 to decimal.
 (ii) Convert hexadecimal 3B2 to decimal.
 (iii) Convert the long binary number 1001001101010001 to octal and to hexadecimal.

Solution:

(i) $(756)_8$

$$= 7 \times 8^2 + 5 \times 8^1 + 6 \times 8^0 = 448 + 40 + 6 = (494)_{10}$$

(ii) $(3B2)_{16}$

$$= 3 \times 16^2 + 11 \times 16^1 + 2 \times 16^0 \quad (\text{put } B = 11)$$

$$= 768 + 176 + 2 = (946)_{10}$$

(iii)

$$\begin{array}{cccccccccccc} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ \hline & & \underbrace{1} & & \underbrace{1} & & \underbrace{1} & & \underbrace{5} & & \underbrace{2} & & \underbrace{1} & & & & & & \end{array}$$

$$= (111521)_8$$

and

$$\begin{array}{cccccccccccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ \hline & & \underbrace{9} & & \underbrace{3} & & \underbrace{5} & & \underbrace{1} & & & & & & & & & & \end{array}$$

$$= (9351)_{16}$$

- Q2** Show the value of all bits of a 12-bit register that holds the number equivalent to decimal 215 in
 (a) binary (b) binary coded octal (c) binary coded hexadecimal and (d) binary coded decimal.

Solution:

- (a) Binary

$$(215)_{10} = (11010111)_2$$

In a 12-bit register, it will be stored as: "0 0 0 0 1 1 0 1 0 1 1 1"

- (b) Binary Coded Octal

$$(215)_{10} = (0327)_8 = 000 \ 011 \ 010 \ 111$$

- (c) Binary Coded Hexadecimal

$$(215)_{10} = (0D7)_{16} = 0000 \ 1101 \ 0111$$

- (d) Binary Coded Decimal

In binary coded decimal, each decimal (0 to 9) digit is represented by 4-bit binary code.

$$(215)_{10} = 0010 \ 0001 \ 0101$$

| | | |
|---|-----|---|
| 2 | 215 | |
| 2 | 107 | 1 |
| 2 | 53 | 1 |
| 2 | 26 | 1 |
| 2 | 13 | 0 |
| 2 | 6 | 1 |
| 2 | 3 | 0 |
| | 1 | 1 |

- Q3** Consider the addition of numbers with different bases

$$(x)_7 + (y)_8 + (w)_{10} + (z)_5 = (k)_9$$

If $x = 36$, $y = 67$, $w = 98$ and $k = 241$, then z is

Solution:

$$\begin{aligned} (36)_7 &= (27)_{10} ; (67)_8 = (55)_{10} ; (98)_{10} = (98)_{10} \\ (z)_5 &= (z)_5 \\ (241)_9 &= (199)_{10} \end{aligned}$$

$$(z)_5 = (199)_{10} - (27)_{10} - (55)_{10} - (98)_{10} \quad \begin{array}{r} 5 \mid 19 \mid 4 \\ \hline 3 \end{array}$$

$$(z)_5 = (19)_{10}$$

$$(z)_5 = (34)_5$$

$$\therefore z = 34$$

Q4 (a) Represent the 8620 into following codes:

- (i) BCD (ii) Excess-3 (iii) 2421

(b) Find 7's complement of the given number $(2365)_7$

Solution:

(a) (i) Write binary equivalent of each decimal

$$8620 \Rightarrow 1000 \ 0110 \ 0010 \ 0000$$

(ii) **Excess-3:** For excess 3, add 3 (binary 0011) to each BCD part.

Hence,

$$\begin{array}{cccc} 1000 & 0110 & 0010 & 0000 \\ +0011 & +0011 & +0011 & +0011 \\ \hline 1011 & 1001 & 0101 & 0011 \end{array}$$

(iii) 2421: It is a weighted binary code

These codes are minor image from the given dotted line.

As $(4)_{10}$ and $(5)_{10}$ make complementary pair.

Similarly $(3)_{10}$ and $(6)_{10}$ make the complementary pair.

Hence, 1110 1100 0010 0000.

(b) For a value/number having a base of r ,

then r 's complement = $(r - 1)$'s complement + 1

Hence, 7's complement of $(2365)_7 = 6$'s complement + 1

$$\begin{array}{r} 6 \ 6 \ 6 \ 6 \\ - 2 \ 3 \ 6 \ 5 \\ \hline 4 \ 3 \ 0 \ 1 \quad \text{6's complement} \\ + 1 \\ \hline 4 \ 3 \ 0 \ 2 \quad \text{7's complement} \end{array}$$

| Decimal digit | 2 | 4 | 2 | 1 |
|---------------|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 1 | 0 |
| 3 | 0 | 0 | 1 | 1 |
| 4 | 0 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 | 1 |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 0 | 1 |
| 8 | 1 | 1 | 1 | 0 |
| 9 | 1 | 1 | 1 | 1 |

Q5 Perform the following conversions:

- (i) $(3287.5100098)_{10}$ into octal (ii) $(675.625)_{10}$ into hexadecimal (iii) $(A72E)_{16}$ into octal

Solution:

(i) To convert $(3287.5100098)_{10}$ into octal:

- Integer part conversion,

$$\begin{array}{r} 8 \mid 3287 \\ \hline 8 \mid 410 - 7 \\ \hline 8 \mid 51 - 2 \\ \hline 8 \mid 6 - 3 \\ \hline 0 - 6 \end{array} \quad \uparrow \quad (3287)_{10} = (6327)_8$$

- Fractional part conversion,

$$0.5100098 \times 8 = 4.0800784 \rightarrow 4$$

$$0.0800784 \times 8 = 0.6406272 \rightarrow 0$$

$$0.6406272 \times 8 = 5.1250176 \rightarrow 5$$

$$0.1250176 \times 8 = 1.0001408 \rightarrow 1$$

$$(0.5100098)_{10} = (0.4051...)_8$$

So, $(3287.5100098)_{10} = (6327.4051...)_8$

(ii) To convert $(675.625)_{10}$ into hexadecimal:

- Integer part conversion,

$$\begin{array}{r|l} 16 & 675 \\ \hline 16 & 42 - 3 \\ \hline 16 & 2 - A \\ \hline & 0 - 2 \end{array} \quad \uparrow \quad (675)_{10} = (2A3)_{16}$$

- Fractional part conversion,

$$0.625 \times 16 = 10.000 \rightarrow A$$

$$(0.625)_{10} = (0.A)_{16}$$

So, $(675.625)_{10} = (2A3.A)_{16}$

(iii) To convert $(A72E)_{16}$ into octal:

- Hexadecimal to binary conversion,

$$(A72E)_{16} = (1010\ 0111\ 0010\ 1110)_2$$

- Binary to octal conversion,

$$\begin{aligned} (1010\ 0111\ 0010\ 1110)_2 &= (001\ 010\ 011\ 100\ 101\ 110)_2 \\ &= (123456)_8 \end{aligned}$$

So, $(A72E)_{16} = (123456)_8$

Q6 If $X = 111.101$ and $Y = 101.110$ calculate $X + Y$ and $\left. \begin{array}{l} X - Y \\ Y - X \end{array} \right\}$ by 2's complement method.

Solution:

Given $X = 111.101$
 $Y = 101.110$

Now
$$\begin{array}{r} X + Y = 111.101 \\ \quad 101.110 \\ \hline 1101.001 \end{array}$$

For
$$\begin{aligned} X - Y &= X + 2\text{'s complement of } Y \\ &= 111.101 + 010.010 \end{aligned}$$

Discard the carry = $\overset{\textcircled{1}}{0}001.111$ as number will be positive
$$\begin{aligned} &= 001.111 \\ &= 1.111 \end{aligned}$$

For
$$\begin{aligned} Y - X &= Y + 2\text{'s complement of } X \\ &= 101.110 + 000.011 \\ &= 110.001 \end{aligned}$$

\therefore There is no carry generated its a negative number.

\therefore Difference = $-(2\text{'s complement of } 110.001) = -1.111$

Q7 Perform the following addition and subtraction of excess-3 numbers:

(i) $0100\ 1000 + 0101\ 1000$ (ii) $1100\ 1011 - 0100\ 1001$

Check the results obtained, by performing the above operations in decimal format.

